

NEW GEOGEBRA FEATURES FOR CONVERGED STUDENT PROJECTS

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Abstract. GeoGebra – software to support scientific and educational activities in the field of STEAM. Its developers are constantly expanding the capabilities of this program. The first version of the program was released in 2002. It was intended to support the study of algebra and planimetry as well as to reveal the connections between them. Since 2014, GeoGebra has supported creating and researching 3D models, and in 2017, two new features appeared: creating augmented reality and 3D printing.

This article will discuss how to use these new features of GeoGebra to support converged projects. The main feature of such projects is the need to integrate the knowledge, methods and tools of several academic subjects to solve the project problem. The authors of the article were interested in the question whether GeoGebra can help students integrate the necessary knowledge and overcome the difficulties associated with lack of knowledge. To get an answer to this question, a pedagogical experiment was conducted. Students of grade 7 were asked to reproduce an exhibit of

the Museum of entertaining Sciences, which represented the brachystochrona problem.

To solve the problem and understand popular science texts, students did not have enough knowledge of mathematics and physics, but they coped with the task. They used computer modeling of the Museum exhibit, as well as the ability to experiment with the model in virtual and real space. They were even able to improve the exhibit. Our experiment suggests that it is not enough to find convergence points in the school curriculum in order to implement the convergent approach. It is also necessary to provide students with tools that enable the integration of knowledge.

Key words: *convergence education, converged student projects, GeoGebra, the brachystochrona problem*

Introduction

Convergent education is an international perspective, according to the participants of the international seminar “Global perspectives in convergent education” Herr, Akbar, Brummet, Flores, Gordon, Gray & Murder, 2019 [1]. The need to develop a convergent approach in education arose due to the increased number of complex scientific, technical and socio-economic problems, the solution of which requires deep integration of the existing scientific knowledge and methods.

The converged approach is included in the list of the 10 best ideas for future investments by the US National Science Foundation (NSF) in 2016. In the European Commission’s “Foreseeing the New Technological Wave” 2004 [2], cognitive technologies were recognized as the first major research initiative of the XXI century.

This direction is actively developed in Russia by the Kurchatov Institute. Its leading experts have prepared a draft concept of “Strategy for the development of converged technologies”. The document notes that a change in the approach to education is an important condition for the development of convergent technologies in science. In the education system of the capital city, the convergent approach is being developed in the project “Kurchatov Center for Continuous Convergent (Interdisciplinary) Education” (Kurchatov Project) was launched. It started in 2011. The convergent education (at the secondary school level) contributes to the development of students’ pre-professional competencies – future specialists aiming at solving interdisciplinary problems in the field of NBICS (nano-, bio-, info-, cogno-, socio humanitarian) technologies. Within the framework of the project, the experimental schools were equipped with the laboratory and research complexes of the Kurchatov Institute. Scientists

of the Institute supervise convergent-oriented projects of the students. These projects are aimed at solving practical problems. The key event is the annual pre-professional conference “Kurchatov project – from practice to experience, from practice to result”. An example of the convergent technologies in education is STEM (S – science, T – technology, E – engineering, M – mathematics) → STEAM (STEM “+” A – arts) → STREAM (STEAM “+” R – reading and writing).

Problem statement and methods

The main difficulty of the convergent-oriented educational projects is to create conditions that make it easier for students to move from the perception of a problem within a single academic subject to a convergent one. We assumed that such a condition could be GeoGebra. This is a fast-developing software product. In 2002, its first version was released. It was intended to support the study of algebra and planimetry, as well as to visualize the relationships between these sections of mathematics.

In 2014 GeoGebra began supporting the study of stereometry. And in 2017, two new features appeared: the creation of augmented reality and support for 3D printing. Now GeoGebra is presented by developers as a program supporting the STEAM education.

We suggested that GeoGebra can be a good tool for students to solve the problems of convergent-oriented projects. As a focus group, we selected 5 students of the 7th grade who were not motivated to study STEAM. We organized an excursion for them to the Museum of entertaining science “Experimentarium” and offered them to choose an exhibit to reproduce in the school environment. They chose the exhibit “The longer the faster”, which demonstrates the brachistochrone problem (Fig. 1). During the excursion the students were introduced to the problem of finding the curve of the fastest descent. It had the following wording: “Given two points A and B lying in the vertical plane. What is the trajectory of a point moving only by gravity from the point A to the point B in the shortest time?” The students learned that the first hypothesis that the brachistochrona is a cycloid was expressed by Johann Bernoulli in 1696 [3]. However, it was not possible to introduce the students to the considerations that led the scientist to this conclusion. To understand these considerations, you need knowledge of optics, which students will only get in grade 11.

The students learned that Johann Bernoulli organized a competition among the most famous mathematicians of the time to find a mathematical proof of this fact. However, the students could not get acquainted with the obtained solutions, since this requires knowledge of variational methods. Its learning goes beyond the school curriculum. The students could conduct an experiment and

make sure that the ball that moves on the cycloid comes to the finishing line earlier than the ball that moves on a straight slide. The effect that this exhibit demonstrated contradicted the knowledge of students that they received in math and physics classes, which are limited only by knowledge of the laws of uniform motion. This was the reason for their interest in this exhibit.



Figure 1. The exhibit of the museum “Experimentarium”

The students were assigned the following tasks:

- 1) recreate the exhibit using computer simulation of the object in GeoGebra and 3D printing;
- 2) add an informational poster to the exhibit;
- 3) prepare your own excursion for students of the same age.

The students could ask for help from any specialists from the proposed list: a physics teacher (S), a computer science teacher, a technology teacher (T); an engineer (E); in the field of science popularization (A); a mathematics teacher (M); a history teacher, a Russian language teacher (H). During the experiment, we recorded the specialists that students wanted to contact for help, students' questions, and their independent progress in solving the project task (Tabl.1).

Table 1. Stages of solving the project problem

№	Stages	Indicators
1.	A theoretical model of the exhibit was created	The types of curves that scientists hypothesized about were determined, and their formulas were found.
2.	A primary computer model of the main elements of the exhibit was created.	Constructed in GeoGebra: points A and B, a straight line, semi-arc cycloids, and possibly other curves passing through these points, a 3D model of the base is constructed.
3.	Computer verification of 3D	The models were converted to STL format

	printing of the main elements of the exhibit was performed.	and checked in the Repetier-Host V2.1.6 program.
4.	A prototype of the exhibit was printed and its critical analysis was carried out.	A prototype of the exhibit was printed on a 3D printer. An experiment was conducted. A list of necessary modifications to the model has been compiled.
5.	An updated computer model of the main elements of the exhibit was created.	The main elements of the exhibit are constructed in GeoGebra, taking into account the comments.
6.	A working model of the exhibit was printed and tested.	Printed on a 3D printer and assembled the exhibit. An experiment was conducted. Disadvantages of data visualization are revealed. The purpose of the computer experiment is determined.
7.	The exhibit is supplemented with elements that improve the visualization of the progress and results of the experiment.	A dynamic 3D computer model was created that combines with the exhibit in augmented reality and allows conducting a computer experiment.
8.	The information poster has been prepared	A poster has been prepared that tells Museum visitors about the purpose and use of the exhibit.
9.	Prepared text of the excursion.	Prepared the text of the excursion, including available explanations of the phenomenon.

Progress of the experiment

Stage 1. Students found on the Internet and in the printed literature [4], [5], following information:

- a physical method for obtaining a cycloid is the trajectory of moving the point on the rim of a wheel of radius r .
- determination of the semi-arc of the cycloid: "the part of the cycloid enclosed between its vertex and the nearest return point»;
- equation of semi-arc cycloids in Cartesian coordinates:

$$x = r \cdot \arccos \frac{r-y}{r} - \sqrt{2ry - y^2}, \quad (1)$$

where $A(0; 0)$ – the vertex; $B(\pi r; 2r)$ – the return point.

Stage 2. Students easily found a way to build a segment and arc of a circle defined by points $A(0, 2r)$ and $B(\pi r, 0)$ using GeoGebra tools. They obtained an image of a semi-arc cycloid by entering equation (1) through the input bar of the software (Fig. 2). Tools of the GeoGebra Classic online did not allow students to

convert this curve to a descent curve, since these tools do not work with implicit curves. The students asked their math teacher for help. To solve this problem, the teacher introduced students to the basic rules of graph transformation (ahead learning of the material of grade 9).

As a result, the formula (1) was reduced to the form

$$x = r \cdot \arccos \frac{y-r}{r} - \sqrt{2r(2r-y) - (2r-y)^2}. \quad (2)$$

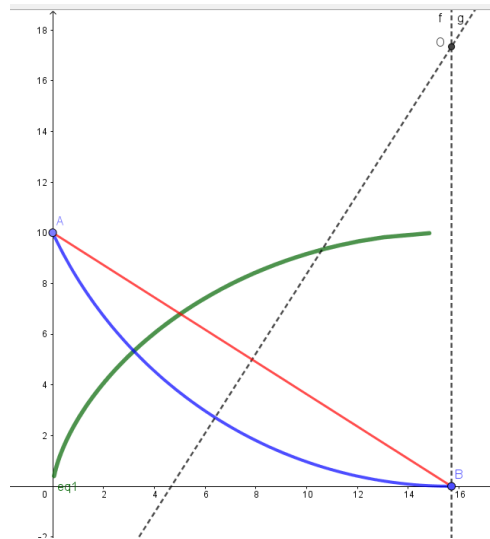
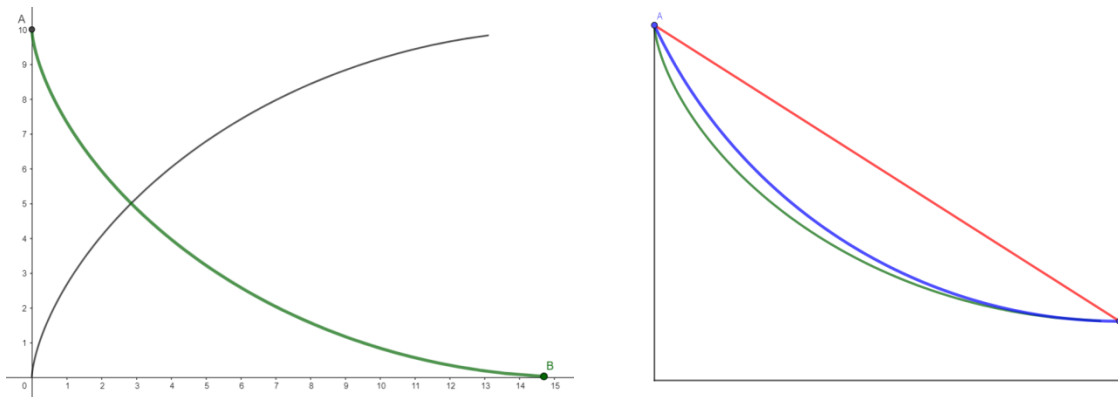


Figure 2. The constructing in GeoGebra

A model of the descent curve on the cycloid AB (Fig.3a) and a frame model of three slides (Fig.3b) are obtained.



a) The transformation of the cycloid AB into A'B' b) The skeleton model

Figure 3. The transformation of the cycloid AB into A'B'

The next difficulty was associated with the creation the fundament of the model. Again, the students were helped by a math teacher. He introduced students to the ways of setting prisms (ahead learning of the material of grade 11). As a result, the students built the model shown in Figure 4.

Stage 3. At this stage, students had difficulty deciding on the choice of parameters for converting the GeoGebra model to STL (“StereoLithography”)

format. The information they needed was contained in an GeoGebrabook, which was written in English. They turned to English and computer science teachers for help.

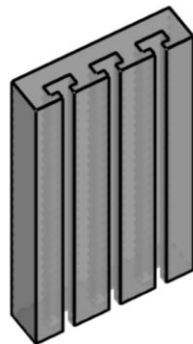


Figure 4. The model of the basis

An English teacher introduced students to the features of technical translation using this e-book as an example (ahead learning of the material of university program). The instructions received by students from the computer science teacher are shown in Figure 5.

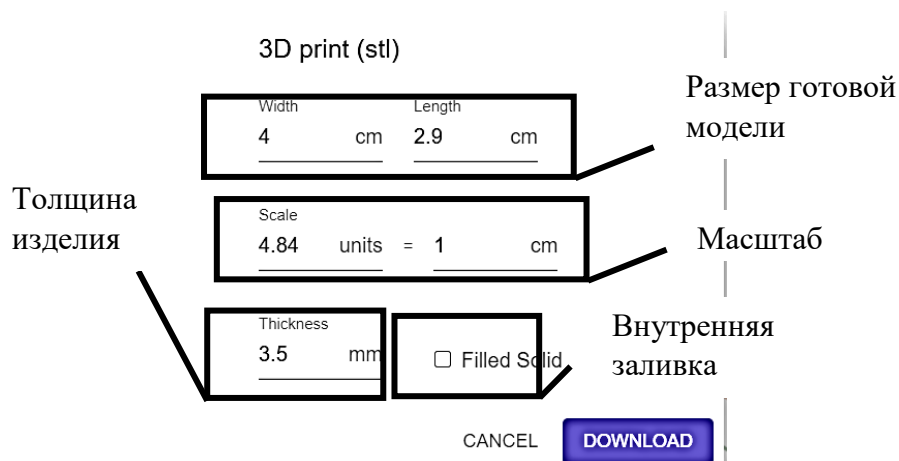


Figure 5. Dialog box STL

He also introduced students to the Repetier-Host V2.1.6 and helped them test the models.

The check showed that implicit curves are not displayed in STL format. The students went back to formula (1) and used the substitution $y \leftrightarrow x$ to get an explicit curve (3) from (1) of the equation):

$$y = r \cdot \arccos \frac{r-x}{r} - \sqrt{2rx - x^2}. \quad (3)$$

Stage 4. After testing the prototype, they made the following judgments:

- to ensure that the slides do not bend, you need to fill the model or create additional props;

- for clarity of experiments it is necessary to increase its duration by increasing the size of the exhibit;
- it is necessary to print two slides in the form of a semi-arch of the cycloid to demonstrate that the cycloid is a tautochronous curve.

Stage 5. Students refined the model and prepared it for 3D printing. The size of the finished model was too large to be printed on a school printer (35 cm in length). For help, they turned to the company ART UP studio, where the exhibit was printed.

Stage 6. As a result, they printed the exhibit shown in Figure 6. To conduct the experiment, it was necessary to select moving objects.



Figure 6. The finished exhibit “the longer, the faster”

The article Sumbatov (2017) [6] helped students make this choice. It highlights the shortcoming of the model the Museum “Experimentarium”: “a physical device that demonstrates clearly the extreme property of a cycloid by means of a heavy ball that rolls along parallel guides curved in the form of a cycloid does not quite demonstrate the classical problem”. The students turned to the physics teacher for an explanation. He introduced students to the material point method (ahead learning of the material of grade 9), the features of the rotational motion of a rigid body (ahead learning of the material of grade 10). As a result, the students decided to use toy cars without wheels to replace spinning with sliding. Experiments with the exhibit have shown that the effect is difficult to observe due to the short duration of movement.

Stage 7. It was decided to Supplement the real experiment with a computer experiment in augmented reality. It was necessary to set the equation of movement of the point along the descent curve for construction a dynamic model. Students asked for help from teachers of physics and mathematics. The physics teacher introduced students to the basics of the dynamics of a material point (ahead learning of the material of grade 9). They discussed with the students the simplest problem of descending a material point $M(x_M; y_M)$ on a straight line AB without friction under the influence of gravity of Earth ($\vec{F}_g = m\vec{g}$). The movement of the point M is represented as a composition of the

movements of its projection on the ordinate axe (M_y) and on abscissus axe (M_x) (Fig. 7). The students came to the conclusion that the movement of the points M_y and M_x is straight and constant accelerated. They move with accelerates $a(M_y) = g \sin^2 \beta$ and $a(M_x) = g \sin \beta \cdot \cos \beta$ respectively.

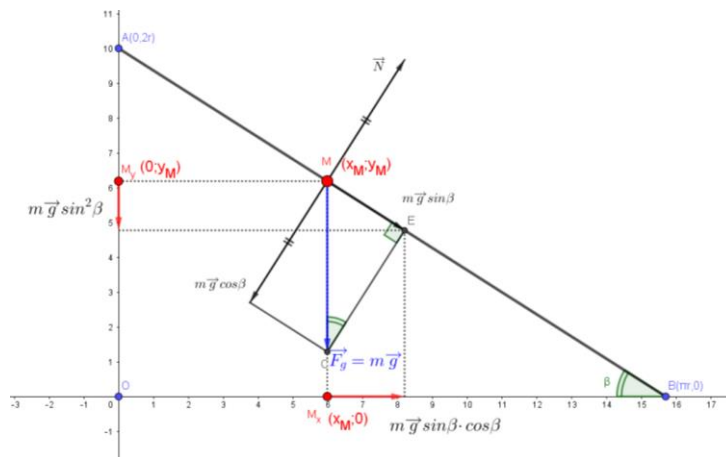


Figure 7. Moving the material point M on the straight line AB

As a result, the dependence of the change in the coordinates of the point M on the time (t) was obtained (4).

$$(4) \begin{cases} x(t) = g \sin \beta \cos \beta \frac{t^2}{2}; \\ y(t) = 2r - g \sin^2 \beta \frac{t^2}{2}. \end{cases}$$

The students tried to transfer the idea of solving this problem to other cases. As the driving force, they considered the projection of gravity on the tangent to the curve at the point M (Fig. 8).

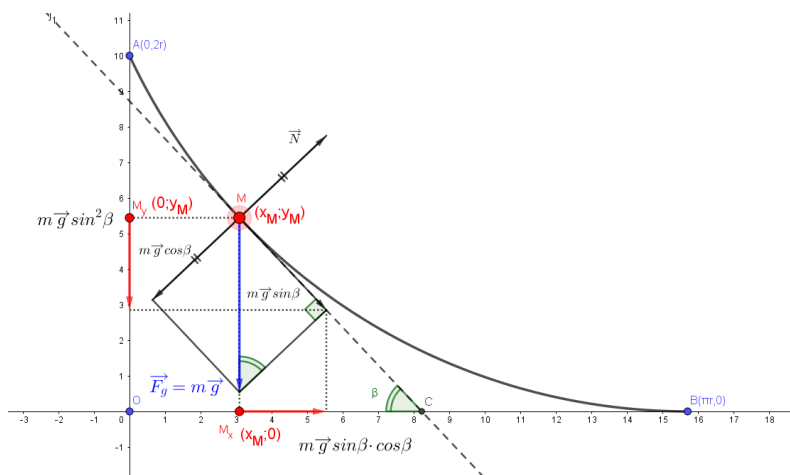


Figure 8. Moving a material point M along the AB curve

A computer experiment convinced them that the angle β is a function of time t . This means that the movements of the points M_y and M_x are not movements with the constant accelerations. They move at variable accelerations

$(M_y) = g \sin^2 \beta(t)$ and $a(M_x) = g \sin \beta(t) \cdot \cos \beta(t)$, respectively. Further deduction of the laws of motion had to be stopped, because it was impossible to get a dependency $\beta(t)$ with the existing knowledge. In addition, the relationship between variable acceleration and displacement was not clear (they need knowledge of mathematical analysis in the framework of the University program). It was decided to use the ready-made dynamic activity and simply transfer it to 3D.

Stage 8. Students chose a program to create the poster. A computer science teacher helped them do this. He suggested them use the software Canva and introduced them to the basics of working in this software. As a result, a poster was created (Fig. 9), which has four sections: “do”, “be surprised”, “explain” and “learn more”.



Figure 9. The information poster

The section “do” contains instructions for conducting experiments: the first leads to the conclusion that the descent on the cycloid is faster than in a straight line; the second – that the descent time from any point of the semi-arc of the cycloid is the same. The section “be surprised” contains a description of the results of experiments. The section “explanation” briefly explains the history of solving the problem, provides brief physical explanations of the phenomenon, and introduces the term “cycloid”. The section “Learn more” offers you to go to the game application LearningApps in order to get acquainted with other properties of the cycloid.

Stage 9. Preparing the tour was an unusual task for students. It seemed to the students that the guides of the Museum of Entertaining Sciences would help them best. They visited the Museum “Experimentanium” again. We made an audio recording of several excursions. By analogy, they wrote the text of their

tour, and then gave it to the guides to evaluate. We received a number of valuable criticisms and suggestions from them. Students chose the optimal time of the excursion, highlighted its entertainment and educational parts. Selected historical facts that would be interesting and understandable to visitors. They included an appeal to the exhibit and poster in their speech. The excursion was held on the day of Convergent education at the opening of the school Museum of entertaining Sciences (Fig. 10).



Figure 10. Students are experimenting with the model

Conclusions

When evaluating the experiment we note:

- students independently coped with the tasks of four stages of the project out of nine;
- they asked for help from 5 specialists in different fields: physics, mathematics, computer science, philology, and popularization of science;
- they came closer to the formulation of problems solved by methods of mathematical analysis, realizing the limits of the possibilities of elementary mathematics and physics.

The experiment confirmed our assumption that GeoGebra is a good tool for solving problems of convergently oriented projects. GeoGebra played a triple role throughout the project:

- 1) acted as an affordable means of implementing project ideas;
- 2) it was a tool that compensates for the lack of theoretical training of students;
- 3) it was a tool whose imperfection leads to the setting of new tasks.

References

- [1] Herr, D., B. Akbar, J. Brummet, et al. Convergence education—an international perspective, *J Nanopart Res* 21, 229 (2019). <https://doi.org/10.1007/s11051-019-4638-7>
- [2] EUR 21357 – Converging Technologies – Shaping the Future of European Societies Luxembourg: *Office for Official Publications of the European Communities* 2004 – 63 pp. (URL: <http://pure.iiasa.ac.at/id/eprint/12590/1/Converging%20Technologies.pdf>)
- [3] *Johann Bernoulli*, (June 1696), Problema novum ad cujus solutionem Mathematici invitantur, *Acta Eruditorum*, **18**: 269. (URL: <https://guest.link/4W5>)
- [4] Tikhomirov, V., The seventh Story. Brachistochrone. Stories about maximums and minimums, library “Quantum”, Russia, 1986, Issue 56, p 192.
- [5] Gindikina, S., Stories about physicists and mathematicians, third edition, expanded, Russia, 2001, p.448.
- [6] Sumbatov, A., *PROCEEDINGS of MIPT*, Russia, 2017, Volume 9, No. 3, pp 66-75.